

Response of Pulsar Braking to Rotation and Internal Structure*

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Pulsar spin-down occurs because of energy and angular momentum loss due to radiative processes. The energy loss equation representing processes of multipolarity n is of the form

$$\frac{dE}{dt} = \frac{d}{dt} \left(\frac{1}{2} I \Omega^2 \right) = -C \Omega^{n+1} \quad (1)$$

where, for magnetic dipole radiation, $C = \frac{2}{3} m^2 \sin^2 \alpha$, $n = 3$, m is the magnetic dipole moment and α is the angle of inclination between magnetic moment and rotation axis. Other multipoles may participate or be important over certain eras of a pulsar's lifetime (for example gravitational radiation in an early era before pulsations are damped). For all such processes the response of the moment of inertia I to the changing angular velocity Ω will produce its effect upon the behavior of $\dot{\Omega}$. While such other processes may be uncertain or speculative, the response of the internal constitution and moment of inertia of a neutron star to changing rotational frequency are not. Taking account of the response, the rate of change of frequency is governed by

$$\dot{\Omega} = -\frac{C}{I(\Omega)} \left[1 + \frac{I'(\Omega) \Omega}{2I(\Omega)} \right]^{-1} \Omega^n \quad (2)$$

where $I' \equiv dI/d\Omega$. This reduces to the usual form quoted in the literature [1] for low frequency or if changes in I are ignored, namely

$$\dot{\Omega} = -K \Omega^n \quad (K = C/I). \quad (3)$$

The usual expression for the moment of inertia in General Relativity [2] is not adequate for our purpose. It ignores the alteration of the metric of spacetime by rotation and the dragging of local inertial frames, and even the centrifugal flattening and changes in internal constitution resulting from the changing density profile of the star with changing rotational frequency. Instead we must use an expression that incorporates these effects as derived by Glendenning and Weber [3].

The dimensionless quantity $\Omega \ddot{\Omega} / \dot{\Omega}^2$ would be equal to the intrinsic index n if the frequency were small or if the moment of inertia were a constant, as can be obtained from (3). However these conditions are not usually fulfilled and the measurable quantity is not the constant index of the energy-loss mechanism (1) but a variable 'braking' index as can be calculated from (2). The measurable quantity is

$$n(\Omega) \equiv \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2} = n - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}. \quad (4)$$

It approaches the index n that characterizes the energy loss mechanism only in the limits $\Omega \rightarrow 0$ or $I \rightarrow \text{constant}$.

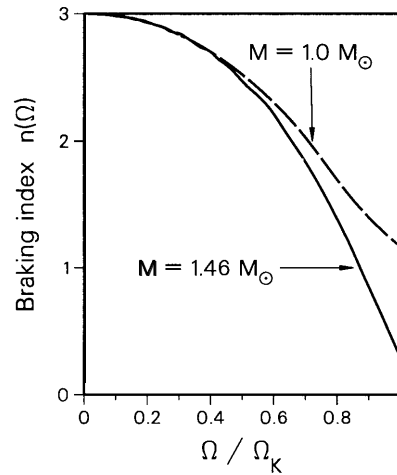


Figure 1: Braking index for two stellar models of different mass showing the strong departure from $n = 3$ for higher frequencies.

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[2] J. B. Hartle, *Astrophys. J.* **150** (1967) 1005.

[3] N. K. Glendenning and F. Weber, *Astrophys. J.* **400** (1992) 647; *Phys. Rev. D* **50** (1994) 3836.